FORMAL LOGIC FOR THE ISLAMIC KNOWLEDGE

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1. INTRODUCTION

The term intelligence is used as a combination of several different abilities – memory, logical reasoning, problem solving, learning through past experience, intuition, self-awareness etc. The intelligence that human being possesses is a natural intelligence. Man finds computer the most suitable to share his intelligence. The intelligence that a computer accepts to think intelligently is the Artificial Intelligence.

Artificial Intelligence is a branch of computer science that deals with ways of representing knowledge using symbols and patterns, rather than numbers, and the heuristic methods for processing information. It is simply a way of making computer thinks intelligently. This branch of computer science is used to solve many types of problems. One of such types is to acquire information that require reasoning. A study of "Formal Logic" techniques and an understanding of arguments and logical reasoning are helpful to solve such type of problems.

Qurān, Hadith, Ijmā and Qiyās are the four fundamental sources of Islamic fiqh (Islamic jurisprudence). These sources can be logically related to each other and if formal logic is applied to the argument based on these sources, then useful information can be obtained.

After applying formal logic methods, computer may be used, not only to access information that are already available but it can deduce many new kinds of information too.

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2. FORMAL LOGIC

Logic is the science of deduction. It aims to provide systematic means for telling whether deduced conclusion does or does not follow from given premises. It is the study of the methods and principles used to distinguish correct from incorrect reasoning.

Logic is about reasoning. It is about the validity of arguments, consistency among statements, and matters of truth and falsehood.

In a formal sense, logic is concerned only with the form of arguments and the principles of reasoning to get true conclusion.

3. CATEGORIES OF A SENTENCE

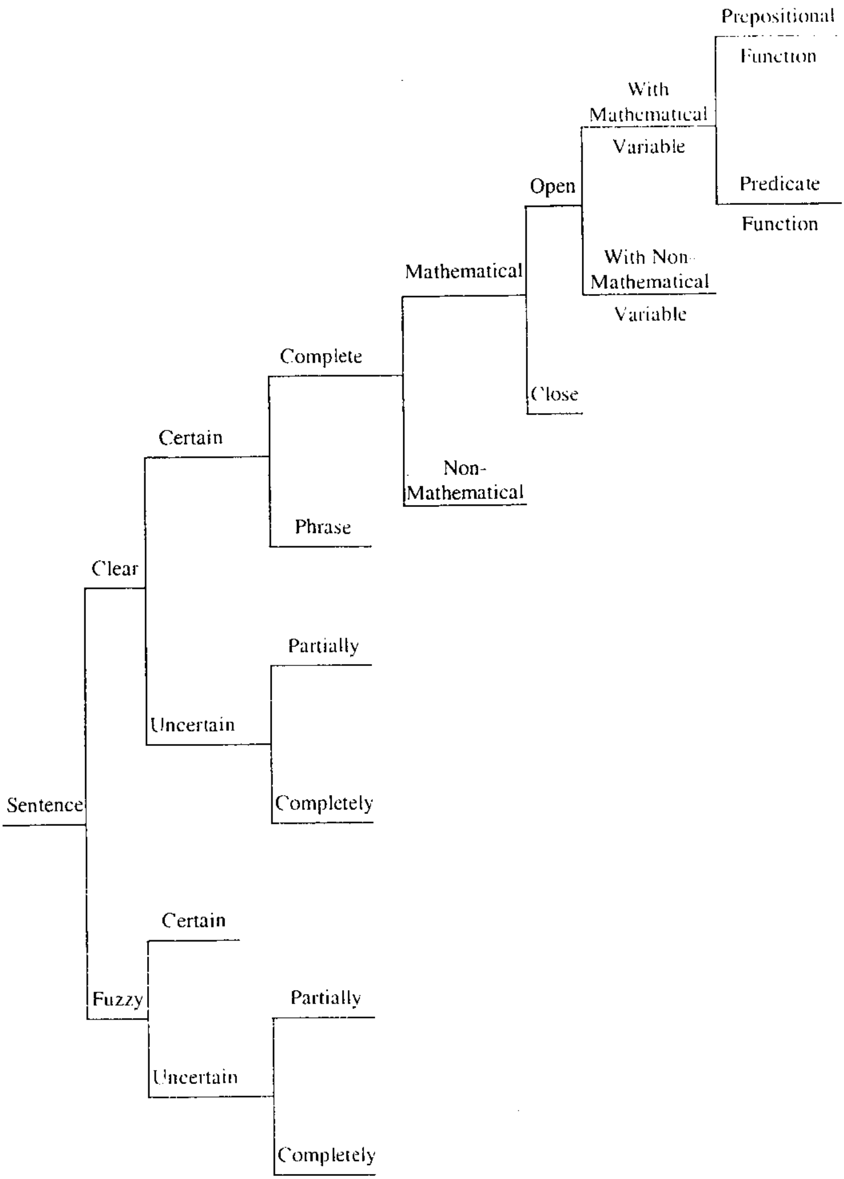
In our daily life, we use different types of sentence. Some of these are commands, some are requests, some are questions, some are statements etc. A sentence may belong to the class of following two categories: (see the tree diagram)

1. Clear
If it is well defined then it is clear.

2. Fuzzy
If it is ill defined then it is fuzzy.
Clear sentence may be divided into:

1.1. Certain
If it is evident then it is certain

1.2. Uncertain
If it is partially or completely hidden then it is uncertain
Certain sentence may be divided into:



Tree Diagram: Categories of a Sentence

1.1.1. Complete

If it contains a complete idea then the sentence is complete

1.1.2. Phrase

If it does not contain a complete idea then it is a phrase e.g. Father of Zarrar, some ayah, fajar and maghrib are all phrases that do not contain a complete idea.

Complete sentence may be divided into:

1.1.1.1. Mathematical

If it can be judged to be true or false then it is mathematical

1.1.1.2. Non-Mathematical

If it can not be judged to be true or false then it is non-mathematical e.g.

Recite the Holy Qurān, or

Are you reciting the Holy Qurān?

For each of the above two sentences, we can not say that it is true or false because the first one is a command and the second one is a question.

Mathematical sentence may be divided into:

1.1.1.1.1. Open
If it contains at least one variable then it is open

1.1.1.1.2. Closed
If it does not contain any variable then it is closed. A closed sentence is also known as a statement e.g.
Saad-bin-Mughira is offering his maghrib's prayer, or
Abu-Ubaidah is reciting the Holy Qurān
are the closed sentences or statements.

Open sentence may be divided into:

1.1.1.1.1.1. Mathematical Function
If it contains a mathematical variable then it is a mathematical function.

1.1.1.1.1.2. Non-Mathematical Function
If it contains a non-mathematical variable then it is a non-mathematical function e.g.
He is going to the Shah Faisal Mosque. Here "He" is a non mathematical variable

Mathematical function may be divided into:

1.1.1.1.1.1.1. Propositional Function
If it contains a subject and a claim in a manner 'x is a sahabi' with one variable x , or '$x+y > 5$' with two variables x and y, where x and y are two real numbers or a claim with more than two variables then it is known as Propositional Function.

1.1.1.1.1.1.2. Predicate Function
If it contains a claim p and objects $o_1, o_2, o_3, \ldots, o_n$, in a manner $p(o_1, o_2, o_3, \ldots, o_n)$ then the claim p itself is known as predicate and the function $p(o_1, o_2, o_3, \ldots, o_n)$ is known as predicate function e.g. in

brother (x,y)
brother is a predicate and x and y are objects. This predicate tells that x is brother of y.

Uncertain sentence that belongs to the class of clear may further be divided into:

1.2.1. Partially Uncertain and

1.2.2. Completely Uncertain
e.g. "Mughira will go to madrasa tomorrow" may be partially or completely uncertain.

Similarly, the sentence that belongs to the class of fuzzy may be divided into:

2.1. Certain
If it is evident then it is certain e.g.
The person whom I met yesterday is muttaqi

2.2. Uncertain
Uncertain fuzzy sentence may be divided into:

2.2.1. Partially Uncertain and
2.2.2. Completely Uncertain
e.g. the fuzzy sentence "Ibn-e-Hazam will go to a big madrasa tomorrow" may be partially or completely uncertain.

4. STATEMENT

A sentence that can be judged to be true or false is called a statement .The truth and falsity of a statement is called its truth-value. A statement is either true or false, but not both, e.g., the statement "Hazrat Abu Bakr Siddique was the muslim caliph" is a true statement while the statement "Maccā is in Pakistan" is false.

5. NEGATION

The negation of a statement is formed by placing the word "not" within the original or the given statement, e.g.,"Kabir is not a muslim" is the negation of the statement "Kabir is a muslim".
In logic , we use a single letter to represent a single complete thought. This means that an entire sentence may be replaced by a single letter of the alphabet, e.g.,
p: Kabir is a muslim
(Here, Kabir is a muslim is replaced by the letter "p")

The negation of the statement "p" is formed by placing a "~"sign before "p", i.e.,

~p: Kabir is not a muslim.

The negation will always have the opposite truth value of the original statement or we may say that it reverses truth value, e.g., if "p" is a true statement then its negation "~p" is a false statement and vice versa. The truth value of "~p" is given by the following table:

P	**~p**
T	F
F	T

Table 1: ~p

6. COMPOUND STATEMENT

Some statements are compound statements, which means that they are composed of sub statements and various connectives (~, ∧, ∨ etc). The fundamental property of a compound statement is that its truth value is completely determined by the truth values of its sub statements together with the way in which they are connected to form the compound statement.

7. CONJUNCTION

In logic, a conjunction is a compound statement formed by combining two simple statements using the word "and". If "p" and "q" represent simple statements, then the conjunction "p and q" is written in symbolic form as "p ∧ q", e.g., if

p: Kabir is a muslim and

q: Kabir lives in Maccā, then

p∧q: Kabir is a muslim and he lives in Maccā.

There are four possibilities for the two statements "p" and "q", two for each, i.e.,

The statement "Kabir is a muslim" is TRUE.
The statement "Kabir is a muslim" is FALSE.
The statement "Kabir lives in Maccā" is TRUE.
The statement "Kabir lives in Maccā" is FALSE.

Conjunctions are true if all of its components are true, and false if even one of its components is false, e.g., the conjunction "Kabir is a muslim and he lives in Maccā " is true only when the statements "Kabir is a muslim" and "Kabir lives in Maccā are both true and is false in all the other cases. The truth value of the compound statement "p∧q" is given by the following table.

P	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Table 2: $p \wedge q$

8. DISJUNCTION

In logic, a disjunction is a compound statement formed by combining two simple statements using the word "or". If "p" and "q" represent simple statements then the *disjunction*
" p or q " is written in symbolic form as " $p \vee q$ ", e. g; if
p: Kabir is a muslim and
q: Kabir lives in Maccā then
$p \vee q$: Kabir is a muslim or he lives in Maccā.

Disjunctions are true if even one component is true, and are false if all components are false, e.g; the disjunction " Kabir is a muslim or he lives in Maccā " is false only when the statements "Kabir is a muslim" and "Kabir lives in Maccā " are both false and is true in all the other cases .The truth value of $p \vee q$ is given by the following table:

P	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Table 3: $p \vee q$

9. PROPOSITION AND TRUTH TABLE

A compound statement P (p, q, r,...) of the sub statements p, q, r, ,... is known as *Proposition,* if the sub statements are variables.

The uses of the connectives within logic are determined by different ways. One simple way is through the use of *Truth Table*. Truth table gives us operational definition of the logical connectives. It is a simple concise way that shows the relationship between the truth-values of a proposition and the truth-values of its variables. The truth table of the proposition **(~p∨q) ∧(~q∨p),** for example, is constructed as follows.

p	**q**	**~p**	**~q**	**~pVq**	**~qVp**	**(~pVq) ^(~qVp)**
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Table4: (~pVq) ^(~qVp)

or precisely:

p	**q**	**(~pVq) ^(~qVp)**
T	T	T
T	F	F
F	T	F
T	F	T

Table 5: (~pVq) ^(~qVp)

10. TAUTOLOGY, CONTRADICTION AND CONTINGENCY

In logic , a tautology is a proposition P(p, q, r,...) that is always true, regardless of truth values assigned to the sub statements p, q, r,.... Similarly, a *contradiction* is a proposition P(p, q, r,...)
that is always false, regardless of the truth values assigned to the sub statements p, q, r,... for example, the

p	q	(p∧q)	~(p∧q)	~(p∧q)∨p
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Table 6: Tautology

proposition ~(p∧q)∨ p is a tautology (Table 6) and the proposition (p∧q)∧~p is a contradiction (Table 7).

p	q	(p∧q)	~ p	(p∧q) ∧~p
T	T	T	F	F
T	F	F	F	F
F	T	F	T	F
F	F	F	T	F

Table 7: Contradiction

Besides tautology and contradiction, there exist a third kind. A proposition, some of whose truth values are true while the remaining are false is known as *contingency*, e. g., $(p \wedge q) \wedge p$ is a contingency (Table 8).

P	**q**	**p∧q**	**(p∧q)∧p**
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Table 8: Contingency

11. LOGICAL EQUIVALENCE

When two propositions P(p,q,r,……) and Q(p,q,r,…..) have the identical truth tables then they are said to be logically equivalent e.g., Consider the truth tables of $\sim(p \wedge q)$ and $\sim p \vee \sim q$ (Table 9 and 10)

p	**Q**	**p∧q**	**~(p∧q)**
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Table 9: $\sim(p \wedge q)$

p	Q	~p	~q	~p∨~q
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Table 10: ~p∨~q

Since the truth tables are the same, the propositions ~(p∧q) and ~p∨~q are logically equivalent and we can write

~(p∧q) ≡~p∨~q

If

p: Kabir is a muslim, and

q: Kabir lives in Maccā, then

~ (p∧q): It is false that Kabir is a muslim and he lives in Maccā

~p∨~q: Kabir is not a muslim or he does not live in Maccā

And according to the equivalence rule, the statement "It is false that Kabir is a muslim and he lives in Maccā " is equivalent to the statement "Kabir is not a muslim or he does not live in Maccā".

12. CONDITIONAL STATEMENT

The statement of the form " if p then q" is called a *conditional statement* and is denoted by

p→q

p is called the *premise*, the *hypothesis*, or the *antecedent* and q is called the *conclusion* or the *consequent*.

The conditional statement p→q is false only when a true antecedent "p" leads to a false consequent "q" and is true in all other

cases, e.g., hakim-Ibn-e- Sina told his patient Hisham, "If you take the medicine, then you will feel better in 24 hours". Here
p: Hisham takes the medicine and
q: Hisham will feel better in 24 hours

Now if Hisham takes medicine and will feel better in 24 hours, then the statement made by hakim Ibn-e-Sina "If you take the medicine, then you will feel better in 24 hours" will be true statement. On the other hand, if Hisham takes medicine and will not feel better in 24 hours, then the statement made by hakim Ibn-e-Sina will be false.

If Hisham does not take the medicine, it is possible that he could feel better or that he could not feel better. In both cases, there is no way to test the statement made by hakim Ibn-e-sina. We cannot say that hakim told Hisham a lie because he told him only what would happen if he did take the medicine.

Since we cannot accuse the hakim of making a false statement to Hisham in these 2 cases, we will say that Ibn-e-Sina's statement is true. The following truth table shows the truth-values of Ibn-e-Sina's statement:

p	**q**	**p → q**
T	T	T
T	F	F
F	T	T
F	F	T

Table 11: p → q

Now consider the truth table of the proposition ~p∨q:

p	**q**	**~p∨q**
T	T	T
T	F	F
F	T	T
F	F	T

Table 12: ~p∨q

Hence $p \rightarrow q \equiv \sim p \vee q$

Now consider the conditional proposition $p \rightarrow q$ and the other simple conditional propositions which contain p and q:
$q \rightarrow p$, $\sim p \rightarrow \sim q$, and $\sim q \rightarrow \sim p$. These propositions are respectively called the Converse, Inverse, and Contra positive of the proposition $p \rightarrow q$. The truth tables of the four propositions are given below:

P	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contra positive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Table 13: Conditional, Converse, Inverse, and Contra positive

13. BICONDITIONAL STATEMENT

The statement of the form "p if and only if q" is called a *biconditional* statement and is denoted by

$$p \leftrightarrow q$$

The biconditional statement $p \leftrightarrow q$ is formed by combining the two conditionals $p \rightarrow q$ and $q \rightarrow p$ under a conjunction "and", i.e.

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

The truth table of the biconditional statement $p \leftrightarrow q$ is:

p	**q**	**p→q**	**q→p**	**p↔q**
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Table 14: p↔q

Thus the biconditional statement p↔q is true when p and q are both true or both false, or in other words, it is true when p and q have the same truth values and is false when p and q have different truth values.

14. ARGUMENT

The relationship of the form

$P_1, P_2, P_3, \ldots P_n \vdash Q$

OR

P_1
P_2
P_3
.
.
.
P_n

Q
is called an *argument*
Argument contains propositions $P_1, P_2, P_3, \ldots\ldots P_n$ known as *premises* and Q which is also a proposition known as *conclusion*.

It says that the proposition Q is concluded from the set of proposition $\{P_1, P_2, P_3, \ldots\ldots, P_n\}$

15. VALIDITY OF AN ARGUMENT

An argument may be valid or invalid. To check the validity of an argument, we try to find whether it contains any *counter example*. A counter example is that, in which the conclusion is false for all of its true premises. The *invalid arguments* are those that have at least one counter example.
Consider the following argument

If Hisham takes the medicine, then he will feel better in 24 hours ($p \to q$)
He takes the medicine (p)
He will feel better in 24 hours (q)
To check its validity, we construct the following truth table:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Table 15: $p \to q, p \vdash q$

Row 1 is the only, that has both the premises ($p \to q$ and p) true and in that row, the conclusion (q) is also true. Since there is no counter example for the given argument, therefore the argument is valid.
Consider another argument

If it rains, Ibn-e-Ubaid will be sick ($p \to q$)
It did not rain ($\sim p$)

Ibn-e-Ubaid was not sick ($\sim q$)
with the following truth table

P	q	p → q	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Table 16: p → q, ~p ⊦ ~q

Row 3 has all the premises (p→q, ~p) true, but the conclusion (~q) is false in that row, i.e., row 3 is a counter example. Thus the argument is invalid. So we conclude that the truth of the two premises (p→q and ~p) does not necessarily imply that the conclusion(~q) is true or "If it rains, Ibn-e-Ubaid will be sick and it did not rain" does not necessarily imply that "Ibn-e-Ubaid was not sick.". This is due to the fact that although, rain caused Ibn-e-Ubaid to sick, but there may be some other incident e.g. Ibn-e-Ubaid ate ice cream, that caused him to sick. On the other hand, the argument

If it rains, Ibn-e-Ubaid will be sick (p→q)
Ibn-e-Ubaid was not sick (~q)

It did not rain (~p)

with the truth table

P	q	p→q	~q	~p
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Table 17: p→q, ~q ⊦ ~p

is a valid argument as the row four is the only, where both the premises are true and in that row, the conclusion is also true. This shows that if Ibn-e-Ubaid was not sick then it is guaranteed that it did not rain. Moreover it is also concluded that there was not a single incident happened that can caused him to be sick.

16. SATISFIABILITY

Arguments whose conclusions are tautology are always valid, regardless of what their premises may be, for a counter example would be a case in which the conclusion is false, and there is no such case if the conclusion is a tautology. Consider for example:

Hanzala is in Mosque (p)
Fārābi is in Mosque or not (qV~q)

with the truth table

p	**Q**	**~q**	**qV~q**
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	T

Table 18: p ⊢ qV~q

Of course, there is no need to work out in the four-case truth table here in order to see that the argument is valid since the conclusion (qV~q) is a tautology, therefore the premise is redundant here.

There are types of arguments for which there is no need to construct a truth table to check their validity e.g., if there is not a single row where all the premises of an argument are true, then according to the definition of *counter example* the argument counts as valid for it has no counter example. The following example gives clear idea

Fārābi is in mosque (p)
Fārābi is not in mosque (~p)

Sun rises in the west (q)
The truth table is:

p	q	~p	p∧~p
T	T	F	F
T	F	F	F
F	T	T	F
F	F	T	F

Table 19: p, ~p ⊢ q

Since none of the rows has the premises (p and ~p) true i.e., the conjunction of the premises is a contradiction, so there is no counter example and therefore the argument is valid.

The contradictory premises "p" and "~p" are said to be jointly unsatisfiable. In general:

"A set of premises is said to be *satisfiable* if there is at least one case in which all members of the set are true i.e., the conjunction of the premises is either a tautology or a contingency, but not a contradiction." The set is said to be unsatisfiable if there is no case in which all members are true, i.e., the conjunction of the premises is a contradiction.

17. SOUNDNESS

Arguments, whose premises form unsatisfiable sets are always valid, according to the, "no counter examples" definition of validity from a contradiction any thing follows but such inferences, although unshakably valid are equally unshakably unsound according to this definition.

"A deductive argument that is valid with all its premises true (i.e. the set of its premises is satisfiable) is said to be sound". A deductive argument is unsound if it is invalid or if the set of its premises unsatisfiable

The following table specifies such situations where an argument is either Sound or Unsound

Conjunction of premises	Conclusion	Satisfiability of the set of premises	Validity of the argument	Soundness of the argument
Tautology	Tautology	Satisfiable	Valid	Sound
Tautology	Contingency	Satisfiable	Invalid	Unsound
Tautology	Contradiction	Satisfiable	Invalid	Unsound
Contingency	Tautology	Satisfiable	Valid	Sound
Contingency	Contingency	Satisfiable	Either	Either
Contingency	Contradiction	Satisfiable	Invalid	Unsound
Contradiction	Tautology	Unsatisfiable	Valid	Unsound
Contradiction	Contingency	Unsatisfiable	Valid	Unsound
Contradiction	Contradiction	Unsatisfiable	Valid	Unsound

Table 20: Soundness of an Argument

It is the sound argument whose conclusions are surely true; mere validity of the inference is not enough to guarantee truth of the conclusion, Nor in validity enough when coupled with mere consistency of the premises, i.e., joint truth of all of them in some possible case. A valid argument does not say that the "derived conclusion " is true but it only says that the "derivation" of the conclusion from the given premises is true. On the other hand, the sound argument says that not only the derivation of the conclusion from the given premises is true but the conclusion itself is also true.

18. SOME VALID ARGUMENTS

The definition of the validity of an argument is provided in section 15. Moreover, it is also explained in the section that how one can check the validity of an argument, using truth table. Some arguments are already given in the section, along with the procedure that determines their validity. Following are some valid arguments whose validity can be +checked. Details are provided for those that need it.

Argument #1

(i) If Hazrat Huzaifah attended A's funeral prayer then A is a non-hypocrite.
Hazrat Huzaifah attended A's funeral prayer
Therefore A is a non-hypocrite.
Symbolically

$p \to q, p \vdash q$

Where
p: Hazrat huzaifah attended A's funeral prayer, and
q: 'A' is a non-hypocrite.

Detail:

Hazrat Muhammad (S.A) provided the list of hypocrites to Hazrat Huzaifah (R.A.). After the death of Hazrat Muhammad (S.A.), if someone died then observers saw whether Hazrat Huzaifah attended the funeral of the person or not. If Hazrat Huzaifah attended the funeral then the observer concluded that the person was not among hypocrites.

OR

ii) If 'A' obeys Rasool-Allah (S.A.) then 'A' obeys Allah.
'A' obeys Rasool Allah (S.A.)
Therefore 'A' obeys Allah
Symbolically

$p \to q, p \vdash q$

Where
p: 'A' obeys Rasool-Allah (S.A). and
q: 'A' obeys Allah

The above two arguments are of the type, $p \to q, p \vdash q$. This type is one of the 'Rules of Inference' which will be discussed latter and is known as *Modus Ponens* (M.P).

Argument #2

If 'A' is a prophet then 'A' is a man of miracle .
'A' is not a man of miracle.
Therefore 'A' is not a prophet

Symbolically

$p \to q\ ,\ \sim q\ \vdash\ \sim p$

Where
p : 'A' is a prophet, and
q : 'A' is a man of miracle

Detail

O Muhammad(S.A),we have sent revelation to you just as we sent to Noah and other prophets after him. We also sent revelation to Ibrahim and Ismaeel and Ishaq and Yaqub and his children and to Eisa and Ayyub and to Yunus and to Haroon and to Suleman and we gave Zaboor to Daood. We also sent revelation to those messengers whom We have already mentioned to you and to those messengers whom We have not mentioned to you; and We spoke directly to Moosa as in conversation(An-Nisā : 163-164)

According to the above ayat and to ijma , Allah sent revelation to all prophets. The argument of the type
$p \to q\ ,\ \sim q\ \vdash\ \sim p$
is known as *Modus Tollen* (M.T)

Argument #3

If Hazrat Ali (R.A) didn't obey shariah in case of deed 'A' then Hazrat Ali (R.A) didn't accept Hazrat Muhammad's (S.A) insult in case of deed 'A'.
If Hazrat Ali (R.A) didn't accept Hazrat Muhammad's (S.A) insult in case of deed 'A' then Hazrat Muhammad (S.A) felt happy.
Therefore if Hazrat Ali (R.A) didn't obey shariah in case of deed 'A' then Hazrat Muhammad (S.A) felt happy.
Symbolically

$$p \rightarrow q, q \rightarrow r \vdash p \rightarrow r$$

Where
p : Hazrat Ali(R.A) didn't obey shariah in case of deed 'A'
q : Hazrat Ali(R.A) didn't accept Hazrat Muhammad (S.A) insult incase of deed 'A'
r : Hazrat M(S.A) felt happy

Detail

When "Muhammad-ur-Rasool Allah" was written at the time of Suleh Hudaibia" the kuffar-e-Maccā raised an objection that they didn't accept Muhammad (S.A) as a Rasool Allah and therefore they asked to erase it and to write "Muhammad ibn-e-Abdullah". Hazrat Muhammad (S.A) asked Hazrat Ali (R.A) to erase it but Hazrat Ali (R.A) refused to do so,. though the order of Hazrat Muhammad (S.A) was shariah. Hazrat Muhammad (S.A) erased it himself and wrote "Muhammad ibn-e-Abdullah". Hazrat Ali (R.A) felt that if he acted according to the shariah then it was an insult of Hazrat Muhammad (S.A). Hazrat Muhammad (S.A) felt happy for this act of Hazrat Ali (R.A).
The argument of the type

$$p \rightarrow q, q \rightarrow r \vdash p \rightarrow r$$

is known as *Hypothetical Syllogism* (H.S)

Argument #4

Either tahajjud prayer is obligatory on a muslim or fast in Ramazan is obligatory on a muslim.
Tahajjud prayer is not obligatory on a muslim.
Therefore fast in Ramazan is obligatory on a muslim.

Symbolically

$p \vee q , \sim p \vdash q$

Where
p: Tahajjud prayer is obligatory on a muslim
q: Fast in Ramazan is obligatory on a muslim

The argument of the type
$p \vee q , \sim p \vdash q$
is known as *Disjunctive Syllogism* (D.S) .

Argument #5

If Hazrat Muhammad (S.A) considered Bait-ul-Maqdis as qiblah then it was revealed to him to do so.
If it was revealed to Hazrat Muhammad (S.A)to consider Bait-ul Maqdis as a qiblah then this revelation is either Qurānic or it is other than Qurān's.
If the revelation is Qurānic then it must be stated in Qurān.
But it is not stated in Qurān
Therefore the revelation is non- Qurānic.

Symbolically

$p \rightarrow q , q \rightarrow r \vee \sim r , r \rightarrow s , \sim s \vdash \sim r$

where

p: Hazrat Muhammd (S.A) considered Bait-ul-Maqdis as Qiblah.
q: It was revealed to Hazrat Muhammad (S.A) to considered to Bait-ul-Maqdis as Qiblah.
r: The revelation is Qurānic.
s: the revelation is stated in Qurān.

Detail

I follow nothing but what is revealed to me (Younūs: 15)

Hazrat Muhammad(SA) considered Bait-ul-Maqdis as qiblah and it had been considered as a qiblah for 17 months. According to the ayat quoted above, prophet follows nothing, except what is revealed to him, i.e. if prophet says something or does something then it is revealed to him to say that or to do so. Now Qurān's revelation says nothing about this consideration, therefore Bait-ul-Maqdis had been considered as a qiblah according to the revelation other than Qurān's.

Argument #6

If and only if Allah speaks to 'A' through revelation (secret instruction) or Allah speaks to 'A' from behind a veil or Allah speaks to 'A' by sending a messenger(an angle),who by His command, reveals whatever He wills then 'A' is prophet.
Allah doesn't speak to 'A' through revelation (secret instruction) and He doesn't speak to 'A' from behind a veil and He doesn't speak to 'A' by sending a messenger (an angel),who by His command, reveals whatever He wills
Therefore 'A' is not a prophet".

Symbolically

$p \vee q \vee r \leftrightarrow s, \sim p \wedge \sim q \wedge \sim r \vdash \sim s$

Where

p: Allah speaks to 'A' through revelation (secret instruction)

q: Allah speaks to 'A' from behind a veil.

r: Allah speaks to 'A' by sending a messenger (an angel), who by His command reveal whatever He wills.

s: 'A' is a prophet.

Detail

Prophet is the only human being, whom Allah speaks. Allah says in Qurān:

It is not the power of any human being that Allah should speak to Him but either through revelation (secret instruction), or from behind a veil, or He sends a messenger (an angel), who by His command, reveals whatever He wills (Ash-Shurā: 51)

According to the above ayat, Allah speaks to a prophet in only the three ways mentioned.

Argument #7

If 'A' recognizes his own entity then 'A' recognizes his RAB.
If 'A' recognizes his RAB then 'A' fears his RAB.
If 'A' fears his RAB then his RAB becomes well pleased with him.
If his RAB becomes well pleased with him then 'A' is successful.
Therefore if 'A' recognizes his own entity then 'A' is successful.

Symbolically
$p \rightarrow q,\ q \rightarrow r, r \rightarrow s, s \rightarrow t \vdash p \rightarrow t$

where,

p: 'A' recognizes his own entity.
q: 'A' recognizes his RAB.

r: 'A' fears his RAB.
s: 'A's' RAB becomes well pleased with him.
t: 'A' is successful.

Detail

Allaḥ says in the Holy Qurān:

Do not be like those who forgot Allah, and Allah caused them to forget their ownselves.(Al-Hashar:19)

i.e., if someone forgets Allah then Allah causes him to forget his own entity. This is a conditional statement. The contrapositive of this statement is " if someone recognizes his own entity then he recognizes his RAB and which is the first premise of the argument 7. The second, third and fourth premises are inferred from the following three ayah:

(i) *The fact is that only those of His servants, who possess knowledge, fear Allah. (Fātir:28)*
(ii) *Allah became well pleased with them and they with Allah. All this is for him who feared his Lord. (Al-Bayyinah:8)*
(iii) *And, above all, they will enjoy Allah's pleasure; this is the supreme success. (At-Tauba:72)*

Now it can not get from the Qurān directly that if someone recognizes his own entity then he is successful but through the reasoning found in the argument #7, it can be concluded.

Argument #8

If arguments contain more than two or three different simple statements as components, it is cumbersome and tedious to use truth tables to test their validity. A more convenient method of establishing the validity of some arguments is to *deduce* their conclusions from their premises by a

sequence of shorter, more elementary arguments that are already known to be valid. Consider, for example, the following argument, in which five different simple statements occur:
Either it is a period of Eid-ul-Azha or if it is a month of Ramazan then fast is obligatory on a muslim.
If sacrifice of animal is not wajib for a muslim, then if fast is obligatory for a muslim, then one who is muttaqi keeps fast
If it is a period of Eid-ul-Azha, then sacrifice of animal is wajib for a muslim.
Sacrifice of animal is not wajib for a muslim.
Therefore, if it is a month of Ramazan, then one who is muttaqi keeps fast

It may be translated into our symbolism as:

$$p \vee (q \rightarrow s)$$
$$\sim r \rightarrow (s \rightarrow t)$$
$$p \rightarrow r$$
$$\sim r$$
$$\therefore q \rightarrow t$$

To establish the validity of this argument by means of a truth table would require a table with thirty-two rows. We can prove the given argument valid, however, by deducing its conclusion from its premises by a sequence of just four arguments whose validity has already been established. From the third and fourth premises, $p \rightarrow r$ and $\sim r$, we validity infer $\sim p$ by *Modus Tollens*. From $\sim p$ and the first premises, $p \vee (q \rightarrow s)$, we validity infer $q \rightarrow s$ by a *Disjunctive Syllogism*. From the second and fourth premises,
$\sim r \rightarrow (s \rightarrow t)$ and $\sim r$, we validity infer $s \rightarrow t$ by *Modus Ponens*. And finally, from these last two conclusions (or sub conclusions), $q \rightarrow s$ and $s \rightarrow t$, we validity infer
$q \rightarrow t$ by a *Hypothetical Syllogism*. That its conclusion can be deduced from its premises using valid arguments exclusively, proves the original argument to be valid. Here the elementary valid argument forms *Modus Ponens* (M.P.), *Modus Tollens* (M.T.), *Disjunctive Syllogism* (D.S.), and

Hypothetical Syllogism (H.S.) are used as *Rules of Inference* by which conclusions are validly deduced from premises.

A more formal and more concise way of writing out this proof of validity is to list the premises and the statements deduced from them in one column, with "justifications" for the latter written beside them. In each case the "justification" for a statement specifies the preceding statements from which, and the Rule of Inference by which, the statement in question was deduced. It is convenient to put the conclusion to the right of the last premiss, separated from it by a slanting line, which automatically marks all of the statements above it to be premises. The formal proof of validity for the given argument can be written as

1. $p \vee (q \rightarrow s)$
2. $\sim r \rightarrow (s \rightarrow t)$
3. $p \rightarrow r$
4. $\sim r$ $\quad / \therefore (q \rightarrow t)$
5. $\sim p$ 3,4, M.T.
6. $q \rightarrow s$ 1,5, D.S.
7. $s \rightarrow t$ 2,4, M.P.
8. $q \rightarrow t$ 6,7, H.S.

Argument #9

If deed 'A' is ordered by Allah then deed 'A' is obligatory and if deed 'B' is prohibited by Allah then deed 'B' is unlawful
Either deed 'A' is ordered by Allah or deed 'B' is prohibited by Allah
Therefore either deed 'A' is obligatory or deed 'B' is unlawful
Symbolically

$p \rightarrow q \wedge r \rightarrow s,\ p \vee r \ \vdash q \vee s$

Where

p: Deed 'A' is ordered by Allah
q: Deed 'A' is obligatory

r: Deed 'B' is prohibited by Allah
s: Deed 'B' is unlawful
The argument of the type
$p \rightarrow q \wedge r \rightarrow s,\ p \vee r \vdash q \vee s$
is known as *Constructive Dilemma* (C.D)

Argument #10

If innovation in religion takes place somewhere then the difference of opinion exists there
Therefore if innovation in religion takes place somewhere then innovation in religion takes place somewhere and the difference of opinion exists there
Symbolically

$p \rightarrow q \vdash p \rightarrow (p \wedge q)$

Where
p: Innovation in religion takes place somewhere
q: The difference of opinion exists there
The argument of the type
$p \rightarrow q \vdash p \rightarrow (p \wedge q)$
is known as *Absorption* (Abs)

Argument #11

No one is deity except Allah and Muhammad (SA) is the messenger of Him
Therefore no one is deity except Allah
Symbolically
$p \wedge q \vdash p$
Where
p: No one is deity except Allah
q: Muhammad (SA) is the messenger of Him

The argument of the type

$p \wedge q \vdash p$

is known as *Simplification* (Simp)

Argument #12

Five times prayer is obligatory on a muslim
Fasting during the month of Ramazan is obligatory on a muslim
Therefore five times prayer is obligatory on a muslim and fasting during the month of Ramazan is obligatory on a muslim
Symbolically

$p, q \vdash p \wedge q$

Where
p: Five times prayer is obligatory on a muslim
q: Fasting during the month of Ramazan is obligatory on a muslim
The argument of the type
$p, q \vdash p \wedge q$
is known as *Conjunction* (Conj)

Argument #13

Hazrat Abu Bakr is muslim's Caliph
Therefore Hazrat Abu Bakr is muslim's Caliph or he was the governor of Iraq
Symbolically

$p \vdash p \vee q$

Where
p: Hazrat Abu Bakr is muslim's Caliph
q: Hazrat Abu Bakr was the governor of Iraq
The argument of the type
$p, q \vdash p \vee q$
is known as *Addition*(Add)

19. THE RULES OF INFERENCE

A *formal proof of validity* for a given argument is defined to be a sequence of statements, each of which is either a premise of that argument r follows from preceding statements by an elementary valid argument, and such that the last statement in the sequence is the conclusion of the argument whose validity is being proved. This definition must be completed and made definite by specifying what is to count as an 'elementary valid argument'. We first define an *elementary valid argument* as any argument that is a substitution instance of an elementary valid argument form. Then, we present a list of just nine argument forms that are sufficiently obvious to be regarded as elementary valid argument forms and accepted as Rules of Inference.

One matter to be emphasized is that any substitution instance of an elementary valid argument form is an elementary valid argument. Thus the argument

$$\sim r \rightarrow (s \rightarrow t)$$
$$\sim r$$
$$\therefore s \rightarrow t$$

is an elementary valid argument because it is a substitution instance of the elementary valid argument form *Modus Ponens* (M.P.). It results form

$$p \rightarrow q$$
$$p$$
$$\therefore q$$

by substituting $\sim r$ for p and $s \rightarrow t$ for q; therefore, it is of that form even though *Modus Ponens* is not *the specific form* of the given argument.
Following is a list of nine elementary valid argument forms that can be used in constructing formal proofs of validity:

Rules of Inference

1. *Modus Ponens* (M.P.)

$p \rightarrow q$
p
$\therefore q$

2. *Modus Tollens* (M.T.)

$p \rightarrow q$
$\sim q$
$\therefore \sim p$

3. *Hypothetical Syllogism* (H.S.)

$p \rightarrow q$
$q \rightarrow r$
$\therefore p \rightarrow r$

4. *Disjunctive Syllogism* (D.S.)

$p \vee q$
$\sim p$
$\therefore q$

5. *Constructive Dilemma* (C.D.)

$(p \rightarrow q) \wedge (r \rightarrow s)$
$p \vee r$
$\therefore q \vee s$

6. *Absorption* (Abs)

$p \rightarrow q$
$\therefore p \rightarrow (p \wedge q)$

7. *Simplification* (Simp.)

$p \wedge q$
$\therefore p$

8. *Conjunction* (Conj.)

p
q
$\therefore p \wedge q$

9. *Addition* (Add.)

p
$\therefore p \vee q$

These nine Rules on Inference are elementary valid argument forms, whose validity is easily established by truth tables. They can be used to construct formal proofs of validity for a wide range of more complicated arguments. The names listed are standard for the most part, and the use of their abbreviations permits formal proofs to be set down with a minimum of writing.

20. THE RULES OF REPLACEMENT

There are many valid truth-functional arguments that cannot be proved valid using only the nine Rules of Inference that have been given thus far. For example, a formal proof of validity for the obviously valid argument

$$p \wedge q$$
$$\therefore q$$

requires additional Rules of Inference.

Now the only compound statements that concern us here are truth-functional compound statements. Hence if any part of a compound statement is replaced by an expression that is logically equivalent to the part replaced, the truth value of the resulting statement is the same as that of the original statement. This is sometimes called the Rule of Replacement and sometimes the Principle of Extensionality.

Any of the following logically equivalent expressions can replace each other wherever they occur:

1. De Morgan's Theorem (De M.): $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$.
   $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$.
2. Commutation (Com.): $(p \vee q) \equiv (q \vee p)$.
   $(p \wedge q) \equiv (q \wedge p)$.
3. Association (Assoc.): $[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$.
   $[p \wedge (q \wedge r)] \equiv [(p \wedge q) \wedge r]$.
4. Distribution (Dist.): $[p \wedge (q \vee r)] \equiv [(p \wedge q) \vee (p \wedge r)]$.
   $[p \vee (q \wedge r)] \equiv [(p \vee q) \wedge (p \vee r)]$.
5. Double Negation (D.N.): $p \equiv \sim\sim p$.
6. Transposition (Trans.): $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$.
7. Material Implication (Impl.): $(p \rightarrow q) \equiv (\sim p \vee q)$.
8. Material Equivalence (Equiv.): $(p \equiv q) \equiv [(p \rightarrow q) \wedge (q \rightarrow p)]$.
   $(p \equiv q) \equiv [(p \wedge q) \vee (\sim p \wedge \sim q)]$.

9. Exportation (Exp.): $[(p \wedge q) \rightarrow r] \equiv [p \rightarrow (q \rightarrow r)]$.
10. Tautology (Taut.): $p \equiv (p \vee p)$.
$p \equiv (p \wedge p)$.

21. PLAYFULLNESS

Atheists some time ask the following type questions to confuse muslims.
"If Allah is Omnipotent then whether He can create a stone that He cannot hold?" suppose the question is asked to a person X(not necessarily a muslim). Now suppose according to X, Allah is not Omnipotent, i.e according to his faith, Allah have not got power over all things and have not absolute control over all affairs. So whatever be the answer of X, it is against the faith of a muslim. But if X is a muslim then according to him, Allah is Omnipotent, i.e, according to him, Allah have power over all things and have absolute control over all affairs. So according to his faith, Allah can create any kind of stone and he can hold any kind of stone. The conjunction of the three statements, i.e "Allah is Omnipotent and he can create a stone and he can not hold it" is a contradiction. Similarly "Allah is Omnipotent and he cannot create the stone" is also a contradiction. The only true conjunction is " Allah is Omnipotent and he can create the stone and he can hold it."

22. PROPOSITIONAL FUNCTION

Let
A = {Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī, Hazrat Umar-bin-Abdul Azīz}

For $x \in A$, consider the following sentence
$p(x)$: x is sahābī
Now $p(x)$ is true for

i. x = Hazrat Abu Bakr
ii. x = Hazrat Umar

iii. x = Hazrat Usmān
iv. x = Hazrat Alī
(since each of the first four is sahābī)
Where as it is false for
x = Hazrat Umar-bin-Abdul Azīz
(since Hazrat Umar-bin-Abdul Azīz is not a sahābī)

Comparing p(x) with the tree diagram given in section #3, we observe that the sentence is

i. Clear
ii. Certain
iii. Complete
iv. Mathematical
v. Open

Moreover, it consists a mathematical variable x therefore p(x) is a propositional function (or an open sentence)
The set A is called the "domain" or "replacement set".
The set {Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī} which is the set of all replacements that changes the propositional function into true sentences is called the solution set or the truth set i.e., if p(x) is a propositional function on a set A, then the set of elements $a \in A$ with the property that p(a) is true is called the truth set T_p of p(x).
In other words,
$T_p = \{x/x \in A, p(x) \text{ is true}\}$
or, simply
$T_p = \{x/p(x)\}$

Notice that a statement consist a truth value (true or false) while a propositional function consists a truth set.

Now consider another set

B = {Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī, Hazrat Khalīd-bin-Waleed}

with three propositional functions

i. p(x): x is sahābī
ii. q(x): x is ashra-e-mubashshirah
iii. r(x): x is died in 20^{th} century

where $x \in B$

Here
p(x) is true for all $x \in B$ i.e. the truth set of p(x) is the set B
q(x) is true for some $x \in B$ and the truth set of q(x) is:
{Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī}
r(x) is false for all $x \in B$ so its truth set is ϕ

Notice, by the preceding example, that a propositional function defined on a set B could be true for all $x \in B$, for some $x \in B$ or for no $x \in B$

23. QUANTIFIER

Let p(x) be a propositional function on a set A, then

$(\forall x \in A)\, p(x)$ or simply $\forall x, p(x)$

is a statement which reads "For every element x belongs to set A. p(x) is a true statement", or simply "For all x, p(x)".
The symbol

$$\forall$$

which reads "For all" or "For every" is called the universal quantifier. Notice that $(\forall x \in A)\ p(x)$ or $\forall x, p(x)$ is equivalent to the set theoretic statement that the truth set of p(x) is the entire set A, that is,

$$T_p = \{x/x \in A, p(x)\} = A$$

Similarly

$$(\exists x \in A)p(x) \quad \text{or simply} \quad \exists x, p(x)$$

is a statement which reads "There exists an element x belongs to set A such that p(x) is a true statement or simply "For some x, p(x)".
The symbol

$$\exists$$

which reads "There exists" or "For some" or "For at least one" is called the existential quantifier. Notice that $(\exists x \in A)\ p(x)$ or $\exists x, p(x)$ is equivalent to the set-theoretic statement that the truth set of p(x) is not empty, that is

$$T_p = \{x/x \in A, p(x)\} \neq \phi$$

Consider
B = { Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī, Hazrat Khalīd-bin-Waleed}
with three propositional functions

iv. p(x): x is sahābī
v. q(x): x is ashra-e-mubashshirah
vi. r(x): x is died in 20^{th} century

where $x \in B$

once again then
i. $(\forall x \in B)\ p(x)$ is a true statement since
$T_p = \{x/ x \in B, p(x)\}$ = { Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī, Hazrat Khalīd-bin-Waleed} = B

Similarly

ii. $(\exists x \in B)\ p(x)$ is also a true statement since

$T_p = \{x/\ x \in B,\ p(x)\}$={ Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī, Hazrat Khalīd-bin-Waleed } $\neq \phi$

iii. $(\forall x \in B)\ q(x)$ is a false statement since

$Tq = \{x/\ x \in B,\ q(x)\}$={ Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī} $\neq B$

where as

$(\exists x \in B)\ q(x)$ is a true statement since

$Tq = \{x/\ x \in B,\ q(x)\}$={ Hazrat Abu Bakr, Hazrat Umar, Hazrat Usmān, Hazrat Alī} $\neq \phi$

$(\forall x \in B)\ r(x)$ is a false statement since
$Tr = \phi \neq A$

Similarly
$(\exists x \in B)\ r(x)$ is also a false statement since
$Tr = \phi$

24. NEGATION OF PROPOSITIONS WHICH CONTAIN QUANTIFIERS

Consider the Ayat *Every soul will taste of death (Āl-e-Imrān: 185)* i.e., "every soul is mortal" which is a proposition. The negation of this propostion is "it is not true that every soul is mortal"; in other words, there exists at least one soul who is not mortal. Symbolically, then, if S denotes the set of souls, then the negation of the proposition can be written as:

$\sim(\forall x \in S)\ (x \text{ is mortal}) \equiv (\exists\ x \in S)(x \text{ is not mortal})$

Furthermore, if p(x) denotes "x is mortal", then the above can be written as:
$\sim(\forall\, x \in S)\; p(x) \equiv (\exists\, x \in S) \sim p(x)$

Whatever be the proposition p(x) the above relation is true i.e. "it is not true that for all x belongs to S, p(x) is true" is equivalent to "for some x belongs to S, the negation of p(x) is true".

Similarly
$\sim(\exists\, x \in S)\; p(x) \equiv (\forall\, x \in S) \sim p(x)$
is true in general i.e.
"It is not true that for some x belongs to S, p(x) is true" is equivalent to " for all x belongs to S, the negation of p(x) is true"

25. TRUTH VALUE OF PROPOSITIONS WHICH CONTAIN QUANTIFIERS

Whenever we want to find the truth value of a statement consisting quantifier, our approach is according to the following:

1. If the statement consists universal quantifier, then we try to find at least one value belongs to the domain set that makes the statement false. That particular value is the counter example. If there exists a counter example then the statement is false. The statement is true only when there is no counter example e.g. let p(x) be a propositional function whose domain set is A. Let for x=a the statement $(\forall x \in A)\; p(x)$ is false then 'a' is a counter example

2. If the statement consists existential quantifier, then we try to find at least one value belongs to the domain set that makes the statement true. If such a value exists then the statement is true. The statement is false only if there is no such value that makes the statement true.

26. RESOLUTION IN PROPOSITIONAL LOGIC

A proof theory is a technique for establishing the validity of arguments. Although, two methods are already discussed, the third one, given below, is the most efficient
This method is as follows:

1. Form the *conflict set* (premises + negation of conclusion)

2. Convert the conflict set to a set of formulae in *clause* form
   (Note: A *literal* is a proposition letter or a proposition letter prefixed by ~.
   Thus b, c, ~d are all literals; a∨b, a∧b and ~~a are not literal.
   A formula is in clause form if it is a literal or a collection of literals all joined by ∨.
   Thus ~p, p∨q, ~p∨~q∨r are all in clause form; p∧q, p→q and ~~p are not.)

3. Repeatedly apply the *resolution rule* described below to try to derive a contradiction.

4. If a contradiction is found then the argument is valid.

Consider the following argument:
If it is a month of Ramazan then fast is obligatory on 'A'
If fast is obligatory on 'A' and 'A' is not sick then 'A' keeps fast
It is a month of Ramazan
'A' is not sick
Therefore 'A' keeps fast

Symbolically

$p \rightarrow q, (q \wedge \sim s) \rightarrow r, p, \sim s \vdash r$
where

p: It is a month of Ramazan
q: Fast is obligatory on 'A'
r: 'A' keeps fast
s: 'A' is sick

To apply the resolution procedure, we perform the following:

1. The conflict set of this argument is:
   $\{p \rightarrow q, (q \wedge \sim s) \rightarrow r, p, \sim s, \sim r\}$
2. Since $p \rightarrow q$ is equivalent to $\sim p \vee q$ and $(q \wedge \sim s) \rightarrow r$ is equivalent to $\sim q \vee s \vee r$, so the conflict set in clause form is:
   $\{\sim p \vee q, \sim q \vee s \vee r, p, \sim s, \sim r\}$

3. We then apply resolution to derive a contradiction:

i.	$\sim p \vee q$	Conflict Set
ii.	$\sim q \vee s \vee r$	
iii.	p	
iv.	$\sim s$	
v.	$\sim r$	
vi.	$\sim q \vee s$	From 2 and 5 by resolution
vii.	$\sim q$	From 4 and 6 by resolution
viii.	$\sim p$	From 1 and 7 by resolution
ix.	Contradiction	From 3 and 8 by resolution

4. We have found a contradiction in the conflict set, and so the argument is valid.

27. PREDICATE

The propositional function (explained in section 22)
p(x): x is sahābī
has two parts. The first part, the variable x, is the subject of the propositional function. The second part - the claim "is sahābī"- referes to a

property that the subject of the statement can have. We can denote the propositional function "x is sahābī" by sahābī (x). In logic this claim is written in short as "sahābī" and is known as "predicate" where as x is known as object and sahābī(x) is known as the predicate function. We can replace x with any element that belongs to the domain set A. If we replace x with Hazrat Abu Bakr i.e. Sahābī (Hazrat Abu Bakr) then it becomes a statement and the statement is true. On the other hand, if we replace x with Hazrat Umar-bin-Abdul Abu then the statement Sahābī(Hazrat Umar-bin Abdul Azīz) is a false statement. The open sentence "Zarrar is a male "can be written in predicate form as male(zarrar).

Similarly, an open sentence with two or three variables can be written in predicate form as:

i. offers (hanzala, prayer)
   i.e., Hanzala offers prayer
ii. keeps (khawla, fast)
   i.e.,Khawla keeps fast
iii. obeys (fārābi, allah)
   i.e.,Fārābi obeys Allah
iv. muallim (talha, saeed, waqas)
   i.e.,Talha is the muallim of Saeed and Waqas, etc.

If we want to write "all of the members of the set B is a sahābī" in predicate form then we have to use universal quantifier. The statement will be:

$(\forall x \in B)$ (sahābī (x))

Similarly, if we want to write "Some of the members of the set B is ashra-e-mubashshirah", then the statement will be:

$(\exists x \in B)$ (ashra-e-mubashshirah (x))

28. RESOLUTION IN PREDICATE LOGIC

The resolution method in predicate logic precedes much as for the propositional logic. Again the stages are:

1. Form the conflict set (premises + negation of conclusion)

2. Convert the conflict set to a set of formulae in clause form

3. Repeatedly apply the *resolution rule* to try to derive a contradiction.

4. If a contradiction is found then the argument is valid.

But there are following two additional tasks that are needed to perform during the resolution procedure.

1. Eliminating existential quantifier and replacing the corresponding variable by either a constant(called a *Skolem constant*) or a function(called a *Skolem function*). This process is called *Skolemization*

2. The resolution rule "step 3", when modified to handle clause form formulae containing variables, requires an extra operation called "unification".

The steps for converting a given sentence into clause form may be described as follows:

Step 1: Convert to *prenex form*

(Note: a formula in the predicate logic in which all the quantifiers are at the front (i.e. have the whole formula with in their scope) is said to be in *prenex form*. For example.

$(\forall x)\,(soul(x) \rightarrow mortal(x))$
$(\forall x)\,(\exists y)\,(likes(x,y))$
are in prenex form where as
$(\forall x)(father(x) \rightarrow (\exists y)(son(y) \wedge loves(x,y)))$
is not)

Step 2: Drive in all negations immediately before an atom. Use for this purpose p instead of (~p) and also use De Morgan's law, i.e., $(\exists x) \sim p(x)$ in place of $\sim(\forall x)\, p(x)$ and $(\forall x) \sim p(x)$ in place of $\sim(\exists x)\, p(x)$.

Using these algorithms the expression
$\exists x\ \forall y\ (\sim\forall z\ \ a(f(x), y, z)\ \forall\ (\exists u\ \ b\,(x, u) \wedge \exists v\ c(y, v))$
is modified as

$\exists x\ \forall y\ (\exists z) \sim a(f(x), y, z) \vee (\exists u\ b(x, u) \wedge \exists v\ c(y, v)))$

Step 3: Rename variables, if necessary, so that all quantifiers have different variable assignments. It should be noted here that the renaming will not change the meaning of the formula because these variables just act as dummies for the corresponding quantifiers.

If we have an expression like

$\forall x\ (\sim p(x)) \vee (\exists y)\ (q(x, y))) \wedge (\forall x)\ p(x) \vee (\forall y)\ (\sim q(x, y)))$

then according to the algorithm of step 3, $\forall x$ quantifier which is in the left most position is retained as it is whereas $(\forall x)\ (p(x) \vee (\forall y)\ (\sim q(x, y)))$ is replaced by the expression $(\forall z)\ (p(z) \vee (\forall w)\ (\sim q(z, w)))$.

Step 4: Purge existential quantifiers. All existentially quantified variables should be replaced by skolem functions, and the corresponding existential quantifiers should be removed.

The skolemization may be understood as follows. If there are existential quantifiers which are preceded by one or more universal quantifiers, i.e. the existential quantifiers are within the scope of universal quantifiers then replace all the existentially quantified variable by a function symbol not appearing anywhere in the expression. For example, in the expression

$\forall v\ \forall x\ \exists y\ p(v, x, y) \rightarrow q(v, y)$

the existential quantifier $\exists y$ is within the scope of the universal quantifiers $\forall v$ and $\forall x$, so according to this algorithm, the skolemized expression is

$$\forall v\ \forall x\ p(v, x, g(v, x)) \rightarrow q(v, h(v, x))$$

This expression is obtained by replacing variable y, which is existentially quantified by operator $\exists y$ in the earlier expression, by the function g(v, x) in left hand side and by h(u, x) in the right hand side of the expression.

Another type of skolemization is that if the existential quantifier does not come within the scope of the universal quantifier then there will not be any functional dependency of existentially quantified variable on universally quantified variable, and hence the existentially quantified variable can be replaced by a constant symbol.

For example, in the following expression

$\exists u\ \forall v\ \forall x\ \exists y\ p(f(u), v, x, y) \rightarrow q(u, v, y)$

the existentially quantified variable u is not within the scope of the universally quantified variable, v, and x, and hence the function f(u) can be replaced b f(a) with "a" being a constant, and the existential quantifier $\exists u$ is removed from this expression. Thus the new expression after this type of skolemization is,

$\forall v\ \forall x\ \exists y\ p(f(a), v, x, y) \Rightarrow q(a, v, y)$

Having understood what is skölemization and applying this algorithm to the expression (3), the skolemized expression become as,

$\forall y\ (\sim a(f(a), y, g(y)) \vee (b(a, h(y)) \wedge c(y)\ k(y)))$

Step 5: Remove all universal quantifiers, since the universally quantified variables are implicitly retained in the expression.
In the event of this step the above expression may now be written as

$(\sim a(f(a), y, g(y)) \vee (b(a, h(y)) \wedge c(y), k(y)))$

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