Quran and Mathematics-I

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ABSTRACT: Quran has not only emphasized on the acquisition of knowledge and laid down foundation of scientific method by making orders of observation , thinking, contemplation, utilization of vision and reasoning for derivation of result but, it also forwarded the greatest step towards knowledge and science by associating performance of its religious rituals with high standard mathematical sciences. For example, Mawaqeet, regulation of lunar colander , determination of direction of Qibla, Ilm ul Faraid,(Inheritance) Islamic decorative art, calculation of Zkat , Ushr, and Kharaj, weight and measures. These Quranic commands and religious necessities compelled and led the muslim scientists to mathematical research which resulted in the invention of numeral and number system. So in this article the author has tried to bring the light on the mathematical aspect of Quran very briefly.

Mathematics & Religious Rituals in Islam

The importance of science and mathematics in Islam can be visualized by the fact that mathematical science is used in the performance of religious rituals in Islam. If we study the history of religions, we can not find a single such like example. There is no any other religion of mankind in which scientific or mathematical concepts and procedures are used in the performance of religious rituals.[1] The Rom Landau recognized the fact by saying that truly startling features of the Arabic achievements in mathematics is the result of its religious commands. It became vitally important for the Muslims to know correctly the position of Makkah in relation to all different parts or locations of the Muslims world. The Muslims were also compelled to determine the movements of sunrise and sunset, the rising and creation of the moon by which the Muslims regulate the observance of the fast of the Ramadan; and finally, the correct measurement of surfaces for the purposes of division of inherited land according to teaching of Quran. In order to perform all these diverse tasks, the Muslims had to developed the mathematical concepts.[2] Besides the Quranic inducement and appreciation of scientific knowledge, there are five main aspects of religious rituals in which mathematics is applied compulsorily;

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1. The regulation of lunar calendar (Its periods are based on the moon).
2. The regulation of the times of five daily prayers (whose periods are based on the sun).
3. The determination of sacred direction or direction of Qibla (whose goal is specific location)
4. The distribution of inheritance (which involves some skills in arithmetic and first degree algebraic equations).
5. The geometry of Islamic decorative art (which involves various geometrical designs and skills).[3]

Umar Farookh concluded that the Muslims turned first of all to the practical sciences - which would yield them an immediate profit, either in private individual life or in religious life of the community such as mathematics, astronomy and medicine. The science of arithmetic was required because it enabled them to calculate inheritances and to prepare calendar to count days and years. From geometry, they could find the direction of Qibla and Hajj routs, and from the astronomy they could determine the beginning of the holy month of Ramadan and the great fasts, and fixed the times for prayers. Hence there were religious base for the Arabs need for sciences.[4] Francoise Micheau concluded that certain mathematical and astronomical learning were necessary to men of law and religion in Islamic civilization. That is why some scientific disciplines were able to find a place in Madrasas besides other institutions. For example, Ilm-ul-Faraid, the science of succession shares, which involves precise juridical rules and complex mathematical processes, was taught in certain religious Madrasas of Islamic civilizations. e.g. in Madrasa Nizamiya of Baghdad, there were two teachers for arithmetic and faraid along with twenty three all. Similarly astronomy was also studied in Madrasas.[5]

Hence we can conclude that Islam incited the Muslims to study and research in the scientific disciplines of knowledge, by basing the religious rituals like, Salat times, Lunar calendar, dates of Hajj and Ramadan, direction of Qibla, law of inheritance, etc. on scientific knowledge. Hence Islam not only incited his followers for science but also his followers were compelled to work out scientific concept for the performance of basic religious obligatory duties like, Salat, Fast of Ramadan, Hajj of Makkah and legacies of Faraid. And it was the greatest impulse given by Islam to science.

Times of Salat and Fasting or time keeping (مواقيت)

i.   Salat (Prayer)

After the faith in the unity of God, punctual observation of five times daily prayers is compulsory and obligatory religious duty of all Muslims.[6] Quran many times ordered for observance of prayers.[7] e.g. (Al-Baqara, 2:110):

وَأَقِيمُواْ الصَّلَاةَ وَآتُواْ الزَّكَاةَ.

"Be steadfast in prayers and regular in charity."[8]

The Quran ordered the Muslim to observe daily prayers at appointed times by saying: (Al-Nisaa', 4:103):

إِنَّ الصَّلَاةَ كَانَتْ عَلَى الْمُؤْمِنِينَ كِتَاباً مَّوْقُوتاً○

"Indeed salat is a prescribed duty that should be performed at appointed times by the believers."[9]

The stated times for prayers are given, with reference to sunrise, sunset, declining of sun, before sunrise, before sunset in different Quranic verses and in Hadith also.[10] As, Quran says, (Al-Qaf, 50:39-40):

وَسَبِّحْ بِحَمْدِ رَبِّكَ قَبْلَ طُلُوعِ الشَّمْسِ وَقَبْلَ الْغُرُوبِ○ وَمِنَ اللَّيْلِ فَسَبِّحْهُ وَأَدْبَارَ السُّجُودِ○

"And celebrate the praises of the Lord, before the rising of the sun and before (its) sitting, and during part of the night, (also) celebrate His praises and (so likewise) after the postures of adoration."[11]

The times of prayers are also prescribed in this verse as: (Al-Isra, 17:78):

أَقِمِ الصَّلَاةَ لِدُلُوكِ الشَّمْسِ إِلَى غَسَقِ اللَّيْلِ وَقُرْآنَ الْفَجْرِ.

"Establish regular prayers - at the sun's decline till the darkness of night, and the morning prayer and reading."[12]

This verse again prescribes times of prayers with reference to sun; as; Zuhr and Asr by saying, "From declining of sun" and Maghrib and Isha by saying, "Till darkness of night."[13] Quran also describes times of prayers in terms of parts of day and night, which are again related to sun; by saying: (Hud, 11:114)

وَأَقِمِ الصَّلَاةَ طَرَفَيِ النَّهَارِ وَزُلَفاً مِّنَ اللَّيْلِ.

"And keep up prayer in the two parts of the day and in the first hours of the night."[14]

There is very comprehensive discussion about the times of prayer in Sura Taha (20:130):

وَسَبِّحْ بِحَمْدِ رَبِّكَ قَبْلَ طُلُوعِ الشَّمْسِ وَقَبْلَ غُرُوبِهَا وَمِنْ آنَاء اللَّيْلِ فَسَبِّحْ وَأَطْرَافَ النَّهَارِ لَعَلَّكَ تَرْضَىٰ O

"And celebrate (constantly) the praise of thy Lord before the rising of the sun, And before its setting; yea, celebrate them for part of the hours of the night, and at the sides of the day: That thou mayest have joy."[15]

The times of prayers, given in books of Ahadith, are also described in similar terms. e.g.

"Abdullah bin Amr reported, God's Messenger as saying, "The time of noon prayer, is when sun passes the meridian and a man's shadow is the same length as his height, so long as the time for the afternoon prayer has not come; the time for the afternoon prayer is as long as the sun has not become yellow; the time of the sunset (maghrib) prayer is as long as the twilight has not ended; the time of the night prayer is up to midnight; and the time of the morning prayer is from the appearance of dawn, as long as the sun has not risen, but the sun rises refrain from the prayer, for it rises between the horns of the devil."[16] (*Sahih Muslim*, Kitab-ul-Salat)

Hence we can conclude that the times of prayers prescribed in Quran and Sunnah, are as follows:

i.   Fajr: Before rising of sun (قبل طلوع الشمس)
ii. Zuhr: Immediately when sun begins to decline the afternoon. In Quranic words, declining of sun (دلوک الشمس) or when sun cross meridian.
iii.   Asr: In the late afternoon (قبل الغروب)
iv.   Maghrib: Immediately after sunset (اطراف النهار)
v.   Isha: End of twilight (غسق الیل، آناءاللیل، زلفًا من اللیل)

And two optional prayers:

i.   Tahajjud: Said at midnight
ii.   Duha: A short before midday.[17]

Thus times of prayers are given in parameters of position of sun or with reference to, length of shadow that is again corresponding to position of sun.[18]

ii. Times of Fasting

Fasting is another obligatory duty of the Muslim as Quran says: (Al-Baqara, 2:183):

يَا أَيُّهَا الَّذِينَ آمَنُوا كُتِبَ عَلَيْكُمُ الصِّيَامُ.

"O Ye who believe! Fasting is prescribed to you."[19]

The time of fasting starts before daybreak and ends after sunset as it is described in the Hadith:

"Ans reported that, the Holy Prophet (A) said "Take a meal (sahri) little before dawn."[20]

There is another hadith about end time of fasting:

"Umar reported that, the Holy Prophet (A) said "When sun approaches on this side and the day retreats on that side and sun sets he who fasts has reached the time to break it."[21] (Muslim, Bukhari)

Hence time of fasting is also given in terms of day break and sunset. Thus according to standard definition of time of prayer and fasting, the interval for Isha and Fajr prayers begin at nightfall and day break, the permitted interval for Zuhr usually begins when sun has crossed meridian i.e. when the shadow of any object has been observed to increase. The interval for Asr begins when shadow increases in twice the length of object to gnomon or sunset and interval for Maghrib and end of fasting starts after sunset. Thus the times of five daily prayers in Islam are defined in terms of astronomical phenominon.[22]

Thus the limits of permitted intervals for prayers are defined in terms of the apparent position of sun in the sky relative to that local horizon. Their times vary throughout the year and are dependent upon terrestrial latitude, when reckoned in terms of meridian other then local meridian, these also dependent upon terrestrial longtitude.[23]

Hence the Muslim, in order to determine the times of prayer and fasting, must know longitude, latitude, process and techniques of their measurements and their use to determine times of prayer. For this purpose, they must know mathematics like, arithmetic, geometry, trigonometry and spherical trignometry. The Muslim scholars invented and used spherical trigonometry to overcome this religious necessity.[24]

Muhammad bin Musa Al-Khawarizmi and other scholars Contribution to Time Keeping or Times of Prayers

Muhammad bin Musa Al-Khawarizmi, first time in history, worked on this topic. He calculated tables for regulating daylight prayers.[25] Al-Khawarizmi wrote two great books on the topic:

i.   *Kitab Sharah fi Tariqa li Ma'arifa Al-Waqat Be Wasatat-il-Shams* (كتاب
شرح فيه طريقة المعرفة الوقت بوساطة الشمس)

ii.   *Kitab Al-Rukhama* (كتاب الرخامة).[26]

The earliest known work on Sundial which deals with the construction of
Sundials is contribution of Al-Khawarizmi. It consists mainly, of a set of tables
of co-ordinates for constructing horizontal Sundials for various latitudes.[27] An
exact from Al-Khawarizmi's table for Sundial construction is occurred in a
treatise on Sundial by Al-Sijzi, which is recently discovered, and present in
Top - Kopi (Turkey) with MS. 3342, 8 + 9.[28] Besides Al-Khawarizmi many
Muslim scholars, meeting with the need of religion for fasting and salat times,
wrote on the time keeping called Ilm-ul-Rukhama like Habsh bin Abduallh Al-
Mazuri (حبش بن عبد الله المزورى) wrote *Kitab Al-Rakhaim wal-Maqabis* (كتاب
الرخائم والمقابيس).[29] Moreover, after Al-Khawarizmi, Habash Al-Hasib also
worked in this field and calculated quantitative estimates of the angle
depression of the sun at nightfall and daybreak.[30] Some well known
books/treatises, on time keepings along with names of their authors, written in
Islamic era, are given as follows:

1. *Risala fil Falak wa Ma'arifat Awqat-is-Salat* (رسالم فى الفلک و معرفة أوقات
الصلوة) by Sharf-ud-Din Musa bin Muhammad Al-Khalili (شرف الدين مو سى
بن الخليلى).

2. *Risala Awqat-is-Salat wa Simit Qibla* (رسالم أوقات الصلوة و سمت قبلم) by Al-
Khalkhani (الخلخانى)

3. *Risail Istikhraj Awqat-is-Salat fi A'maal-il-Falkiyya* (رسائل إستخراج أوقات
الصلوة في أعمال الفلكية) by Muhammad bin Muhammad Al-Makli Al-Andulusi
(محمد بن محمد المكلى الاندلسى).[31]

Many treatises on latitude of sun, longitude at Zuhr and Asr prayers, and
duration of morning and evening twilight accur in several Zijs of medieval
Islam.[32] This discipline is called Ilm-ul-Miqaat in Arabic sciences.

Abu Rehan Al-Bairuni (ابو ريحان البيرونى) also discussed and determined the
times of fasting in his famous book Qanun-e-Mas'udi (قانون المسعودي).[33] After
13th century, time keeping was in the hands of *muwaqqits* (professional
astronomers), employed in the mosques. The tables available to the muwaqqits
had to be used together with instruments which could establish when the given
present time had arrived. The main source of regulating the daylight prayers
was available to the Muslims in the form of Sundials.[34] Al-Khalili, Ibn Yunus

etc. were famous *muwaqqits*. However, we will discuss the details in the next section under the title: Time Keeping and Sundials.

Direction of Qibla (سمت القبلة)

The Ka'ba is the most famous sanctuary of Islam called Baitullah, situated at the centre of great mosque at Makkah.[35] The Muslim all over the world are enjoined to face the sacred direction of Qibla during their prayers[36] prescribed by Quran as: (Al-Baqara, 2:144):

فَوَلِّ وَجْهَكَ شَطْرَ الْمَسْجِدِ الْحَرَامِ.

"Now shall we turn thee to a Qibla that shall please thee. Turn the face in the direction of sacred mosque."[37]

The words of tradition of Holy Prophet (A) are:

إذا قمت إلى الصلوة, فاسبغ الوضوء ثم استقبل القبلة.

"When you stand for prayer, first make Wudu then stand facing Qibla." (*Sahih Muslim*, Kitab-ul-Salat)[38]

The mosques are oriented towards the Qibla, the mehrab or prayer niche in mosques indicate direction of Qibla. Islamic tradition further prescribes that person performing certain acts like recitation of Quran, announcing call to prayer (Azan), the ritual slaughter of animal for food, should stand in the direction of Qibla.[39] Thus Muslims have been spiritually and physically oriented towards Kaba and Makkah and hence the direction of Qibla has fundamental importance in their lives.[40]

Consequently Muslim must know the direction of Qibla to assist organization of various aspects of religious rituals in Islam. But the calculation of direction of Qibla was problem of purely geographical mathematics and its determination was not possible without the measurement of geographical co-ordinates and the direction of computation of one locality from another by procedures of geometry, trigonometry and spherics.[41]

Thus Muslims were bound to inquire into geography, mathematics, geometry and spherical trigonometry, to determine direction of Qibla for religious purposes. As a result of this religious inducement, many Muslim scholars devoted their attention to this problem, one of them was Al-Bairuni. He wrote a book on this subject, called *Tahdid-ul-Amakin* (The determination of co-ordinates of cities).[42] Al-Bairuni also wrote a comprehensive treatise on

the subject called, *Fi Tariq-ul-Asnai li-Ma'arifa Simit-e-Qibla Waghera* (The methods for determination of direction of Qibla etc.) in his Encyclopedia book Qanun-e-Mas'udi. He discussed the topic beautifully and gave the mathematical formulas.[43] Al-Bairuni also wrote a book called *Al-Risala fi Ma'arifat Simit-e-Qibla*.[44] Besides Al-Bairuni, Saif-ud-Din Abd-ul-Haq wrote a book called *Al-Ma'rifa bi Dalail Qibla* (المعرفة بدلائل قبلة). Other famous works of Muslim scientists on the topic of *jihat al-Qibla* are given:

i. *Kitab Tahdid-ul-Qibla* (كتاب تحديد القبلة) by Ibn Al-Marakushi.[45]

ii. *Risala Awqat-is-Salat wa Simat Qibla* (رسالہ اوقات الصلوة وسمت قبلہ) by Al-Khalkhani

iii. *Risala fi Tahqiq Jihat Qibla* (رسالہ فى تحقيق جهت قبلة) by Shaikh Baha-ud-Din Aamli

iv. *Simat-ul-Qibla li-Aafaq-il-Farisiyya* (سمت القبلہ لآفاق الفارسية) by Razi-ud-Din Muhammad Al-Khazini[46]

v. *Kitab-uz-Zij Al-Majisti* by Al-Batani.[47]

vi. *Simat-ul-Qibla bil Hisab* (سمت القبلة بالحساب) (*Determination of Direction of Qibla by Arithmetic*), *Risala Sharh fiha Ittijah-ul-Qibla* (رسالہ شرح فيها اتجاه القبلہ), *Risala Buhitha fiha Kayfiyyat Istikhraj Simat-ul-Qibla fi Jami' Inha-il-Aalam* (رسالہ بحث فيها كيفية استخراج سمت القبلہ فى جميع انحاء العالم) (*Treatise about the extraction of direction of Qibla in whole world*) by Ibn-ul-Haitham.[48]

vii. *Kitab Simat-ul-Qibla* (كتاب سمت القبلة) (*The Book on Direction of Qibla*) by Al-Naziri.[49]

viii. Ibn Al-Banna Al-Marakushi wrote on the subject along with in other book with the title *Kitab Tahdid-ul-Qibla* (كتاب تحديد القبلة).[50]

Ibn Yunus, Abu Al-Wafa, Ibn-ul-Haitham, and Al-Khalili, etc. also made a worth seeing contribution to the subject and Baqi Yazdi prepared a Qibla-centered world map.[51]

Calendar and Taqweem

The basic obligatory religious duties in Islam like Hajj,[52] fasting,[53] and various festivals throughout the twelve months, are related with lunar calendar. Lunar month starts with first appearance of lunar crescent. The Muslim start fasting with first appearance of lunar crescent of Ramadan, which is ninth month of lunar year, till end of it.[54] Abu Huraira reported God's messenger as saying:

"Calculate on the basis of new moon of Shaban when Ramadan begins."[55] (Tirmidhi)

Similarly, Hajj is performed at Makkah from 8th to 13th of Dhul Hijja, twelfth month of lunar calender.[56] Thus it was religious compulsion of Muslims to establish and formulate lunar calendar more accurately, to perform their religious duties. And Quran directly guides Muslims in this regard by saying: (Al-Baqara 2:189):

يَسْأَلُونَكَ عَنِ الأَهِلَّةِ قُلْ هِيَ مَوَاقِيتُ لِلنَّاسِ وَالْحَجِّ.

"They asked thee concerning the new Moons. Say: They are but signs to mark fixed period of time in (the affairs of) men, and for pilgrimage."[57]

Here Quran is describing that crescent is for time keeping (*Mawaqeet*) and determination of time of Hajj, people will calculate and determine length of time or duration of time for their lives that is called calendar. Quran again describes the fact more comprehensively. (Yunus, 10:5)

هُوَ الَّذِي جَعَلَ الشَّمْسَ ضِيَاء وَالْقَمَرَ نُوراً وَقَدَّرَهُ مَنَازِلَ لِتَعْلَمُواْ عَدَدَ السِّنِينَ وَالْحِسَابَ.

"It He who made the sun to be shining glory and the moon to be a light, and measured out stages for it, that ye might know the number of years and the count (of time)."[58]

Here Quran is referring towards stages of moon for calculation of calendar. The calendar can not be calculated, with high accuracy, with respect to sun. The time of year (length of time interval) with respect to sun can not be divided completely in days, and 12 months and fraction of time is rested. Roman made 13 months.[59] But Quran indicates about the number of months as 12: (Al-Tawba, 9:36):

إِنَّ عِدَّةَ الشُّهُورِ عِندَ اللّهِ اثْنَا عَشَرَ.

"The number of months in the sight of Allah is twelve."[60]

Shaykh Tantawi Jawhari writes in his commentary on Quran, called *Al-Jawahir fi Tafseer-il-Quran-il-Karim*, that this verse contains many astronomical and mathematical facts. It requires very deep and wisdomful research.[61] The concepts given in this verse are not our subject here. Hence these concepts and requirements bind the Muslims to calculate lunar calendar. The lunar calendar as prescribed by Quran is made of 12 months and each month having 29 or 30 days.[62]

The moon revolves around earth in elliptical orbit with varying speed of rotations and completes its trajectory in 29 or 30 days with total period of rotation around earth 354/48/34 days.[63] Thus, in order to prepare Islamic calendar, the Muslim had to know the orbit of moon, its speed, trajectory around earth, for the visibility of moon in advance, in the evening of twenty-ninth day at any locality. But may parameters are involved in this process - celestical co-ordinates of the sun and moon, apparent relative speed of these two luminaries; latitude of place etc.[64] In order to meet with this problem, the Muslim had to know arithmetic, geometry, spherical trigonometry and spherical astronomy etc.

Authorships of Muslim scientists on taqweem

Uraib bin Sa'd Al-Qatib Al-Qurtubi wrote a beautiful treatise on topic, called *Tafseel-uz-Zman wal-Anba*,[65] which is translated in French by R. Dozy. Muhayyi-ud-Din Al-Maghribi wrote a book, called *Kitab Al-Qanun Al-Tarheel-ish-Shams wal-Qamar fi Manazil wa Ma'rifa Awqat-il-Layl wan-Nihar* (كتاب القانون الترحيل الشمس و القمر فى المنازل و معرفة اوقات الليل و النهار).[66] Similarly, Nasir-ud-Din wrote *Kitab Istikhraj-ut-Taqweem* (كتاب استخراج التقويم),[67] And Muhammad bin Musa Al-Khawarizmi wrote *Kitab-ul-Zij*.[68] Muhammad bin Musa Al-Khawarizmi also wrote two other great books on this topic, called (i) *Kitab Sharh fihi Tariqat-ul-Ma'rifa al-Waqt bi Wasatat-ish-Shams* (كتاب شرح فيه طريقة المعرفة الوقت بوساطة الشمس) (*Discussion on method of determination of time with reference to the sun*). (ii) *Kitab-ul-Rukhama* (كتاب الرخامة).[69] A calendar was compiled for Al-Hakim II, this gives the data when sun inters the twelve signs of the zodiac according to Sindhind and Ashab-al-Mumtahan. Hence we have been able to confirm that this refers to the Zij of Al-Khawarizmi.[70] Habash-al-Hasib wrote *Zij-ul-Dimashqi*, with great care of calculation. Thabit worked on visibility of moon and two his treatises are present on the subject, for visibility by calculation and by table.[71]

Al-Bairuni wrote one chapter on "day," month,[72] year and their history, and on "calender,"[73] in his book *Kitab-ul-Tafheem*. Al-Bairuni also wrote two books on the subject of determination of Islamic calender and moon called (i) *Kitab Royat-al-Ahilla* (كتاب رويت الاهلة), (ii) *Kitab Manazil-ul-Qamar* (كتاب منازل القمر).[74] Abu-ul-Fazal Muhandis wrote *Kitab Royat-ul-Ahilla* (كتاب رويت الاهلة) (*Book on the creation of moon*).[75] Similarly Ibn Al-Majdi wrote *Kitab Khulasat-ul-Aqwal fi Ma'rifat-il-Waqt wa Royat-il-Hilal* (كتاب خلاصة الاقوال فى معرفة الوقت ورويت الهلال).[76]

Other important treatises are:

Al-Zij Al-Sabi (الزيج الصابى) by Al-Batani, *Al-Zij Al-Hakimi Al-Kabir* (الزيج الحاكمى الكبير) by Ibn Yunus, *Kitab Al-Haya* by Muhayyi-ud-Din Al-Urdi, *Tadhkira fi Ilm-ul-Hayya* by Tusi, *Zij Yalkhani* (زيج يلخانى) by Nasir-ud-Din Tusi, *Zij Ibn Shatir, Zij Abu Ma'shar Balkhi* and *Zij Mamuni* by Hubaish bin Abdullah, etc.[77] Thabit bin Qurra wrote *Al-Risala fi Royat-il-Ahilla min Jadawal* (الرسالة فى رويـة الأهلة من جداول) and *Kitab Royat-il-Ahilla bil Janub* (كتاب رويـة الأهلة بالجنوب).[78]

Law of Inheritance (Ilm-ul-Faraid)

Law of inheritance (Ilm-ul-Faraid) is main portion of Islamic law in which shares of heirs of a deceased Muslim are determined from the property of a deceased Muslim with certain ratios and conditions, given in Quran & Sunnah.[79] The Holy Prophet (A) described that Ilm-ul-Faraid is one third of all useful knowledge.[80] Quran describes about the stated portions of both men and women with their parents and kindred leaves.[81] (Al-Nisa, 4:7):

لِّلرِّجَالِ نَصِيبٌ مِّمَّا تَرَكَ الْوَالِدَانِ وَالأَقْرَبُونَ وَلِلنِّسَاء نَصِيبٌ مِّمَّا تَرَكَ الْوَالِدَانِ وَالأَقْرَبُونَ مِمَّا قَلَّ مِنْهُ أَوْ كَثُرَ نَصِيباً مَّفْرُوضاًO

"From what is left by parents and those nearest related there is a share for men and a share for women, whether the property be small or large - a determinate share."[82]

Quran also discussed ratio of shares with detail in different circumstances or conditions. (Al-Nisa, 4:11-12):

يُوصِيكُمُ اللّهُ فِي أَوْلاَدِكُمْ لِلذَّكَرِ مِثْلُ حَظِّ الأُنثَيَيْنِ فَإِن كُنَّ نِسَاء فَوْقَ اثْنَتَيْنِ فَلَهُنَّ ثُلُثَا مَا تَرَكَ وَإِن كَانَتْ وَاحِدَةً فَلَهَا النِّصْفُ وَلأَبَوَيْهِ لِكُلِّ وَاحِدٍ مِّنْهُمَا السُّدُسُ مِمَّا تَرَكَ إِن كَانَ لَهُ وَلَدٌ فَإِن لَّمْ يَكُن لَّهُ وَلَدٌ وَوَرِثَهُ أَبَوَاهُ فَلأُمِّهِ الثُّلُثُ فَإِن كَانَ لَهُ إِخْوَةٌ فَلأُمِّهِ السُّدُسُ مِن بَعْدِ وَصِيَّةٍ يُوصِي بِهَا أَوْ دَيْنٍ آبَآؤُكُمْ وَأَبناؤُكُمْ لاَ تَدْرُونَ أَيُّهُمْ أَقْرَبُ لَكُمْ نَفْعاً فَرِيضَةً مِّنَ اللّهِ إِنَّ اللّهَ كَانَ عَلِيما حَكِيماًO وَلَكُمْ نِصْفُ مَا تَرَكَ أَزْوَاجُكُمْ إِن لَّمْ يَكُن لَّهُنَّ وَلَدٌ فَإِن كَانَ لَهُنَّ وَلَدٌ فَلَكُمُ الرُّبُعُ مِمَّا تَرَكْنَ مِن بَعْدِ وَصِيَّةٍ يُوصِينَ بِهَا أَوْ دَيْنٍ وَلَهُنَّ الرُّبُعُ مِمَّا تَرَكْتُمْ إِن لَّمْ يَكُن لَّكُمْ وَلَدٌ فَإِن كَانَ لَكُمْ وَلَدٌ فَلَهُنَّ الثُّمُنُ مِمَّا تَرَكْتُم مِّن بَعْدِ وَصِيَّةٍ تُوصُونَ بِهَا أَوْ دَيْنٍ وَإِن كَانَ رَجُلٌ يُورَثُ كَلاَلَةً أَو امْرَأَةٌ وَلَهُ أَخٌ أَوْ أُخْتٌ فَلِكُلِّ وَاحِدٍ مِّنْهُمَا السُّدُسُ فَإِن كَانُوْا أَكْثَرَ مِن ذَلِكَ فَهُمْ شُرَكَاء فِي الثُّلُثِ مِن بَعْدِ وَصِيَّةٍ يُوصَى بِهَا أَوْ دَيْنٍ غَيْرَ مُضَآرٍّ وَصِيَّةً مِّنَ اللّهِ وَاللّهُ عَلِيمٌ حَلِيمٌO

"Allah thus directs you as regard your children's (inheritance): to the male a portion equal to that of two females: If only daughters, two or more, their share is two-third of the inheritance; if only one, her share is a half. For parents, a sixth share of the inheritance to each, if deceased left children; if no children and the parents are the (only) heirs, the mother has third; if the deceased left brothers (or sisters) the mother has sixth. (The distribution is all cases is) after the payment of legacies and debts. Ye know not whether your parents or your children are nearest to you in benefits. These are settled portion ordained by Allah; and Allah is All-Knowing, All wise. If what your wives leave, your share is half, if they leave no child; but if they leave a child, ye get a fourth; after payment of legacies and debts. In what you leave, their share is fourth, if you leave no child; but if you leave a child, they get an eighth; after payment of legacies and debts. If the man or woman whose inheritance is in question, has left neither ascendants nor descendants but has left a brother or a sister, each one of two gets a sixth; but if more than two, they share in a third; after payment of legacies and debts; so that no loss is caused (to anyone). Thus it is ordained by Allah; and Allah is all knowing, most forbearing."[83]

Shares of "kalala": Distant kindreds are also given in Quran, (Al-Nisa, 4:176):

يَسْتَفْتُونَكَ قُلِ اللَّهُ يُفْتِيكُمْ فِي الْكَلَالَةِ إِنِ امْرُؤٌ هَلَكَ لَيْسَ لَهُ وَلَدٌ وَلَهُ أُخْتٌ فَلَهَا نِصْفُ مَا تَرَكَ وَهُوَ يَرِثُهَا إِن لَّمْ يَكُن

لَّهَا وَلَدٌ فَإِن كَانَتَا اثْنَتَيْنِ فَلَهُمَا الثُّلُثَانِ مِمَّا تَرَكَ وَإِن كَانُوا إِخْوَةً رِّجَالاً وَنِسَاء فَلِلذَّكَرِ مِثْلُ حَظِّ الْأُنثَيَيْنِ يُبَيِّنُ اللَّهُ لَكُمْ أَن

تَضِلُّوا وَاللَّهُ بِكُلِّ شَيْءٍ عَلِيمٌ O

"They ask thee for a legal decision. Say: Allah directs (thus) about those who leave no descendants or ascendants as heirs. If it is a man that dies, leaving a sister but no child, she shall have half the inheritance: If (such a deceased was) a woman, who left no child, Her brother takes her inheritance: If there are two sisters, they shall have two-thirds of the inheritance (between them): if there are brothers and sisters, (they share), the male having twice the share of the female. Thus doth Allah make clear to you (His law), lest ye err. And Allah hath knowledge of all things."[84]

There are many other Quranic verses related to law of inheritence.[85]

Hence according to Holy Books, Siraj-ud-Din Sajawandi writes, shares are six: Half, fourth, third, fifth, sixth and made half and multiple according to conditions.[86] Thus Quranic rules for distribution of estates of a deceased Muslim to various relatives, are complicated and their application calls for some skills in arithmetic and algebraic equations.[87] The calculation of legal shares of natural heirs could be solved by arithmetic of fraction.[88] If estate is a land, then geometry must be known. Thus we can conclude that in order to address religious problem of inheritance, the Muslim should have knowledge of arithmetic, number theory, algebraic equation, roots, measurement, geometry etc. as Ibn Khuldun writes that Ilm-ul-Faraidh is an art of arithmetic. Specific methods and operations are required to determine the shares of inheritance. The whole number, fraction, extraction of roots and Surds, known and unknown quantities, etc. are involved in it. Of course every kind of arithmetic is required in Ilm-ul-Faraidh. Ilm-ul-Faraidh comprises two parts - jurisprudence (Fiqah) and the arithmetic, by which the shares are determined.[89]

Shaykh Tantawi Jawhari writes, in his commentary (exegesis) of Quran, named, *Al-Jawahir fi Tafseer-il-Quran-il-Karim*, about law of inheritance (Ilm-ul-Mirath) and its effects on various branches of knowledge. Tantawi says that Ibn-ul-Haitham prepared a table for calculation of shares of heirs, by which, unknown quantities (numbers) can be determined with help of, known quantities and with certain rules. The art of calculating unknown quantities with the help of known quantities, using common mathematics and its rule, establish and promote following disciplines of knowledge;[90]

1. *Ilm-ul-Hisab-ul-Hawai* (Knowledge of calculating by mind without writing)
2. *Ilm-ul-Hisab wa Takhat* (Knowledge of place-value based number system)
3. *Ilm-ul-Jabr wal-Muqabla* (Algebra and algebraic equations)
4. *Ilm-ul-Hisab-il-Khatain* (Knowledge of calculation of unknown quantities by short, particular methods)
5. *Ilm-ud-Dirham wad-Dinar* (Solution of problems by algebra)
6. *Ilm-ul-Uqood*
7. *Ilm-ul-Ta'abi* (Science of arrangement of armed forces in war)
8. *Ilm-ul-Hisab wan-Nujum* (Science of mathematical astronomy and astrology)
9. *Ilm-ul-Faraid* (Knowledge of kinds of inheritance).

The science of measurement of area of land is also necessary to follow law of inheritance. Hence it promotes science of measurement and geometry. Tantawi concludes: *"The Muslims are how much incited to search treasures of knowledge from Quran."*[91]

Ilm al-Faraid and Invention of Algebra

Al-Khawarizmi's book, *Al-Jabr wal-Muqabala*, was the result of this religious inducement and Al-Khawarizmi acknowledge it by saying, *"I composed a short work on calculating by completion and reduction, such as men constantly require in cases of inheritance, legacies, partition, law suits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computation, and other objects of various sorts and kinds are concerned."*[92]

Al-Khawarizmi's *Al-Jabr wal-Muqabala* comprises two parts, first half is devoted to numbers, their roots, and calculation, mathematical operation, measurement, various types of algebraic equation and demonstrations of the validity of his methods and the 2nd half contains, the solutions of practical problems of inheritance, legacies, measurements, etc. and the examples to show that how the science of arithmetic, of algebra could be applied to the problems posed by the requirements of the Muslim law of inheritence.[93] Al-Khawarizmi also wrote another book on inheritance is called *Kitab-ul-Wasaya* (كتاب الوصايا).[94]

Contributions of Muslim Scientists to Mathematics of Inheritance or *Wasaya* and Measurement of Land or *Masaha*

Many other scholars paid attention to this discipline as Ibn Khaldun quotes Ibn Yunus, *"Religious scholars in Muslim cities have paid much attention to it (inheritance). Some authors are inclined to exaggerate the mathematical side of discipline and to pose problems requiring for their solution various branches of arithmetic, such as algebra, the use of roots, and similar things."*[95]

Number of the Muslim scholars worked out on the topic. e.g. Abu Hanifa Al-Dainuri (ابو حنيفة الدينورى) wrote *Kitab-ul-Wasaya* (كتاب الوصايا).[96] Abu Abdullah Muhammad bin Al-Hassan (ابو عبد الله محمد بن الحسن) wrote *Kitab-ul-Wasaya* (كتاب الوصايا) and *Kitab Hisab-ul-Wasaya* (كتاب حساب الوصايا).[97] Al-Hassan bin Ziyada Al-Lolwi (الحسن بن زياده اللولوى) wrote *Kitab-ul-Faraid* (كتاب الفرائض) and *Kitab-ul-Wasaya* (كتاب الوصايا),[98] and Ahmad bin Umar Al-Shaybani (احمد بن عمر الشيبانى) wrote *Kitab-ul-Wasaya* (كتاب الوصايا).[99]

Other important books on the topics are:

Kitab-ul-Faraid (كتاب الفرائض) and *Kitab-ul-Wasaya* (كتاب الوصايا) by Muhammad bin Ahmad Al-Tahawi (محمد بن احمد الطحاوى),[100] *Kitab-ul-Wasaya fil Hisab* (كتاب الوصايا فى الحساب) by Dawud bin Khalaf Al-Asfahani (داود بن خلف الاصفهانى);[101] *Kitab-ul-Faraid* (كتاب الفرائض) and *Kitab-ul-Wasaya* (كتاب الوصايا) by Safwan bin Yahya (صفوان بن يحى),[102] *Kitab-ul-Wasaya* (كتاب الوصايا) by Abu Yusuf Al-Masisi (ابو يوسف المصيصى) and *Kitab-ul-Wasaya* by Al-Karabisi (الكرابيسى),[103] *Kitab-ul-Wasaya wal- Hisab-ud-Daur* (كتاب الوصايا والحساب الدور) by Abu Ibrahim Al-Muzni (ابو ابراهيم المزنى);[104] *Kitab-ul-Faraid* (كتاب الفرائض) by Abu Al-Hassan Ali bin Muhammad Al-Misri (ابو الحسن على بن محمد المصرى),[105] *Kitab-ul-Faraid* (كتاب الفرائض) and *Kitab-ul-Wasaya* (كتا ب الوصايا) by Ya'qub bin Ibrahim (يعقوب بن ابراهيم),[106] and *Kitab-ul-Faraid* (كتاب الفرائض) by Al-Naziri (النزيرى).[107] Similarly many other scholars like; Ibn Al-Sairfi (ابن الصيرفى), Sufiyan Al-Suri (سفيان الثورى), Abd-ul-Rahman bin Ziad (عبد الرحمن بن زياد), Yahya bin Aadam (يحيى بن آدم) and Yazid bin Harun (يزيد بن بارون) wrote books with the title *Kitab-ul-Faraid* (كتاب الفرائض).[108] Ibn-ul-Haitham wrote *Risala fi Ilaqat-il-Jabr bi Ilm-il-Faraid* (رسالة فى علاقة الجبر بعلم الفرائض)[109] and Tusi (طوسى) wrote *Kitab fil Mirath* (كتاب فى الميراث).[110]

Moreover, the measurement of areas of land should also be known to the Muslims, to act upon the Islamic law of inheritance. Number of Muslim scholars worked on the subject of measurement of areas of land. It is called Ilm-ul-Masahat (علم المساحت) in Islamic civilization. Al-Khawarizmi wrote Bab-ul-Masaha (باب المساحة) in his book *Al-Jabr wal-Muqabala* (الجبر و المقابلة) on the topic.[111] Many other Muslim scientists worked on the subject of Masaha but we can not describe the names of all of them here. Some wellknown works are: *Kitab-ul-Masaha* (كتاب المساحة) by Ibn Nahiya (ابن ناحية), *Kitab-ul-Masaha* (كتاب المساحة) by Abu Barza (ابو برزه), *Kitab-ul-Masaha wal-Handasa* (كتاب مساحة والهندسة) by Abu Kamil (ابو كامل),[112] *Kitab Masahat-ul-Halqa* (المساحة والهندسة) by Al-Karabisi (الكرابيسى),[113] *Kitab fi Masahat-il-Ashkal* (كتاب فى مساحة الحلقه) by Thabit bin Qurra (ثابت بن قرة),[114] *Risala Masahat Ba'd-ul-Astah* (الأشكال) by Al-Karkhi (الكرخى),[115] *Muqaddima fil Masaha* (رسالة مساحت بعض الاسطح) by Umar Khayyam,[116] *Kitab Masahat-ul-Ashkal-il-Basita wal-Kurra* (مقدمة فى المساحة) by Tusi,[117] Risala fil-Masahat (كتب مساحة الاشكال البسيطة والكرة) by Al-Kashi (رسالة فى المساحات),[118] and *Kitab Talkhis-ul-Hisab wal-Jabr wa-A'mal-ul-Masaha* (الكاشى) by Baha-ud-Din Aamli (كتاب تلخيص الحساب و الجبر و اعمال المساحة).[119] Similarly Ibn-ul-Haitham wrote four books on the subject called: *Risala fil Masaha* (بهاؤ الدين عاملى), *Kitab Masahat-ul-Kurrah* (رسالة فى المساحة)

(كتاب مساحة الكرة), *Maqala fi Usul-il-Masaha wa Zikriha bil-Brahin* (فى مقالة) *Kitab fil Masaha ala Jiht-il-Usul* (اصول المساحة و ذكرها بالبراهين) كتاب فى) and (المساحة على جهت الاصول).[120] Al-Hakim Muhammad bin Al-Adli Al-'Ani (الحكيم) wrote *Kitab fil-Masaha* (كتاب فى المساحة).[121] Abu Al-Wafa (محمد بن العدلى العانى) wrote the famous book *Kitab ma Yahtaju Ilaih-il-Ummal wal Kitab min Sana'a-til-Hisab* (كتاب ما يحتاج اليه العمال و الكتاب من صناعة الحساب),[122] Abu Hatim Al-Fazari (ابو حاتم الفزارى) wrote *Muqaddima fil-Hisab* (مقدمة فى الحساب) and Abd-ul-Qahir Al-Baghdadi (عبد القاهر البغدادى) wrote *Kitab fil-Masaha* (فى كتاب المساحة).[123]

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(To be continued)

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